Yet More Evidence for the Emptiness of Plurality*

Alan Bale

MIT, Concordia University

1. Introduction

How are count nouns, mass nouns, plurals, classifiers, and measure nouns interpreted in natural language? Is there a semantic systematicity involved in distinguishing these classes of words and corresponding functional morphemes? In this paper, I hope to make some contribution towards finding an answer to these questions. My focus for now will be narrow. Below I examine the intricacies of how plural marking influences the semantics of measure nouns. As I hope to show, the consequences of this influence shed some light into how nouns in general are represented in a grammar and how plurality interacts with these representations.

Traditional treatments of the plural morpheme as a function that generates groups from a set of singularities encounters three problems. First, such a function over-generates the number of possible groups a denotation can have. This problem comes to the forefront when examining NPs such as *pounds of potato*. Second, it fails to generate large groups when the singular denotation is empty. This problem occurs when certain measure nouns specifying small units of measurement are combined with certain count nouns denoting large objects, as with the phrase *grams of apples*. Third, such a function cannot account for why numerals less than one must combine with plural nouns (*0.75 grams/*gram of saffron*). All three of these problems disappear when the group forming function is eliminated and instead root nouns without number marking are interpreted as being inherently plural (cf. Sauerland, 2003; Sauerland, Anderssen, & Yatsushiro, 2005; Borer, 2005; and Krifka, 1995).

The outline of this paper is as follows. First I review some background concepts concerning the traditional treatment of the plural morpheme. I then present the three prob-

*I would like to thank David Barner, Gennaro Chierchia, Brendan Gillon, Danny Fox, and Irene Heim for their helpful comments and suggestions. This research would not have been possible without grants from the Social Sciences and Humanities Research Council of Canada, grant number 756-2006-0484.

lems such a treatment encounters with respect to measure nouns. Next, I discuss changing the traditional treatment of plurality by eliminating the group-forming function. This change will allow the plural morpheme and numerals to interact with measure nouns. Finally, I demonstrate how this change in the treatment of plurality can be extended to account for common count nouns.

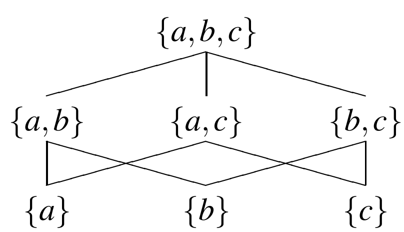
2. Background on Generating Groups

Link (1983) proposed that the plural morpheme should be interpreted as a function that forms a set of pluralities from a set of singularities. A definition of a function similar to Link's is given in (1).

(1) a. $[\![PLURAL]\!] = PL$
b. $PL(X) = \{Y : Y \subseteq X \ \&\ Y \neq \emptyset\}$

Note I have simplified this function slightly for exposition purposes. I have added a set notation and I have allowed the plural morpheme to include atoms in the plural denotation (as in Spector, 2003, 2007), two features that are absent from Link's original proposal.

In a theory of plurality such as (1), nouns (a.k.a, singular nouns) are interpreted as a set of individuals. The plural function generates a set of groups based on this set of individuals. For example, if in a given context the word *student* is interpreted as the set $\{a,b,c\}$, then the plural noun *students* in the same context would be interpreted as the set $\{\{a,b,c\},\{a,b\},\{b,c\},\{a,c\},\{a\},\{b\},\{c\}\}$, where sets with more than one member represent groups and where sets with only one member are treated as being equivalent to singularities.[1] Importantly, groups are ordered with respect to the subgroup (or subset) relation. This ordering is depicted in (2).

(2)


Link's theory of plurality became very influential in Linguistics for mainly two reasons. First, the ordering relation between groups allowed for a simple and unified characterization of the plural and singular definite determiner. By hypothesizing that the such a determiner selects the largest group (with respect to the subgroup relation), not only did Link provide adequate truth conditions, he also explained why such determiners induce a uniqueness and existence presupposition. Second, the plural function used in nouns can be extended to account for distributivity effects in verbs (see Link, 1983, for details).

[1] This treatment of singular sets was first proposed by Quine (1963).

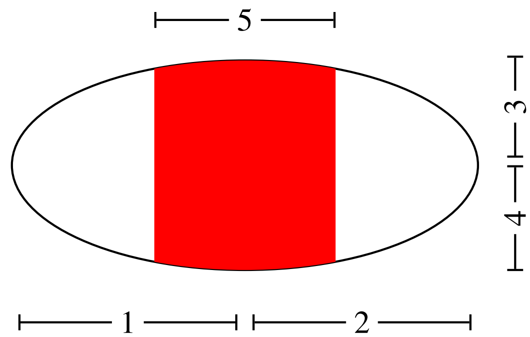


Figure 0.1: A graphical representation of the two pound lump of mashed potato sitting on the table. The vertical strip in the center represents the spilt food dye. The bracket represent the different places pointed to in the order indicated by the numbering.

The benefits of such a theory aside, a potential difficulty arises for Link's account when one considers how plurality interacts with weight measure nouns like *grams* and *pounds*. It is unclear how the plural function would apply to nouns of weight. Straightforward applications run into three empirical problems: the problem of too many pounds, the paradox of grams and the inability for singulars to combine with numbers less than one. In section 3, I discuss each of these problems.

3. Three problems

3.1 The Problem of Too Many Pounds

One problem that arises with measure nouns centers on how many singular items there are in a given situation. The problem is best understood by an example, so let me begin this section by constructing one. Consider a situation where there is a two-pound lump of mashed potato sitting on a table. Suppose that while cooking I accidently spill some red food dye on the potatoes putting a stripe down the center of the lump (see figure 0.1). In this situation I can use demonstratives and finger-pointing to talk about certain portions. For example I could point to the left half of the lump and truthfully utter the sentence in (3a). I could also point to the right half and truthfully utter the sentence in (3b).

(3)
   a. This pound of potato is partially red. (left-half, 1)
   b. That pound of potato is also partially red. (right-half, 2)
   c. Furthermore, this other pound of potato is partially red. (top-half, 3)
   d. Also, that other pound of potato is partially red. (bottom-half, 4)
   e. However, this pound of potato is completely red. (center, 5)

Next I could point to the top half and truthfully utter the sentence in (3c). I could then point to the bottom half and truthfully utter the sentence in (3d). Finally I could point to the center stripe of the lump and truthfully utter the sentence in (3e).

For convenience let's label the different pounds of potato with the numbers 1 through 5. The numbers will indicate the temporal order of the pointing (see figure 0.1). The fact that all four sentences are possible in this situation has some consequences for the denotation of the phrase *pound of potato*. It suggests that 1, 2, 3, 4 and 5 are all members of the singular denotation. In other word, the set $\{1,2,3,4,5\}$ is a subset of the denotation of *pound of potato*.

If all these different pounds are members of the singular denotation, then a Link-style plural function will form lots of different groups. Specifically, since 1 and 2 are members of the singular denotation, the group $\{1,2\}$ must be a member of the plural denotation. Since 3 and 4 are members of the singular denotation, the group $\{3,4\}$ must also be a member of the plural denotation. Furthermore, since 1, 2 and 5 are members of the singular denotation, the group $\{1,2,5\}$ must be a member of the plural denotation. These groups, created as a consequence of the plural function, lead to some interesting and wrong empirical predictions.

Since the plural function creates these groups, minimally the plural denotation of *pounds of potato* will contain these groups, as shown in (4a). With respect to this denotation, the numeral modifiers *two* and *three* simply restrict the denotation to groups that have two members and three members respectively. The denotation of *two pounds of potato* is given in (4b) while the denotation of *three pounds of potato* is given in (4c).

(4) a. $[\![pounds\ of\ potato]\!] = \{..., \{1,2\}, \{3,4\}..., \{1,2,5\}...\}$
b. $[\![two\ pounds\ of\ potato]\!] = \{..., \{1,2\}, \{3,4\}...\}$
c. $[\![three\ pounds\ of\ potato]\!] = \{..., \{1,2,5\}...\}$

The problem with these denotations is that they predict that it would be possible to point at the two pound lump of potato on the table and talk about it as *three pounds of potato*. There is a group, namely $\{1,2,5\}$, that has a cardinality of three. For example, the sentence in (5a) should be coherent in the context specified above.

(5) a. At least three pounds of potato are on the table.
b. I poured red dye on the two pounds of potato.

Not only do the denotations in (4) predict that one can talk about groups of three, they also predict that one cannot use the phrase *the two pounds of potato* to refer to the two pound lump of potato on the table. For example, given the denotation of *two pounds of potato*, the sentence in (5b) should be unacceptable. The definite determiner requires there to be a unique two pound group. However, as shown in (4b) there is no unique two pound group.[2]

[2]Amendments could be made to the theories to deal with one of the problems by prohibiting groups that contain two members that material over-lap with one another. This amendment could be made in the plural function itself by specify that group formation prohibits material over-lap. However, the problem of uniqueness still remains.

Both of these predictions are incorrect. The sentence in (5a) is quite odd in a context where there is a two-pound lump of potato on the table, yet the sentence in (5b) is perfectly acceptable.

3.2 The Paradox of Grams

Another problem for a Link-style analysis of the plural morpheme is encountered when one considers measure nouns whose unit of measurement is small. For example, consider the contrast between the NPs *pounds of apples* and *grams of apples*. Whereas a pound can contain several apples, a gram can only contain bits of an apple. Yet in many contexts the two noun phrases can be used interchangeably. Consider the two sentences in (6).

(6) a. I put the two pounds of apples on the table.
b. I put the 900 grams of apples on the table.

With the knowledge that two pounds is roughly equivalent to 900 grams, the sentences in (6) have similar truth conditions. Each could be used to talk about a two-pound bag of apples that I put on the table.

Although the two NPs can be used to talk about the same object, there is a significant difference in how they refer. The DP *the two pounds of apples* denotes a group that consists of two pounds. In contrast, the DP *the 900 grams of apples* denotes a group that consists of 900 grams. According to a Link-style analysis, these groups were built up from different sets of singularities, one a set of pounds and the other a set of grams. A problem arises for the Link-style analysis when one tries to be explicit about the singular denotation *gram of apples*. In (7), I give one attempt at a possible denotation.[3]

(7) $[\![\text{gram of apples}]\!] = \{x : x \in [\![apples]\!] \;\&\; \mu_g(x) = 1\}$

According to (7) the denotation of *gram of apples* contains all the entities that weigh one gram and are either apples or groups of apples. Yet no apple or group of apples weighs only one gram (an apple weighs about 150 grams). In most contexts, this singular denotation would be empty. However, since the plural denotation is formed from the singular, in most contexts the plural denotation would be empty as well, contrary to the evidence given in (6). The denotation in (7) cannot be the right denotation for the singular *gram of apples*.

As an alternative to (7), consider the denotation in (8).

(8) $[\![\text{gram of apples}]\!] = \{x : x \preceq \iota([\![apples]\!]) \;\&\; \mu_g(x) = 1\}$

The iota-operator picks out the largest group of apples while the $\preceq$ symbol represents the *material-part-of* relation. This denotation of the singular contains all the apple-stuff that weighs one gram. Unlike the denotation in (7), the denotation in (8) is not empty.

[3]For simplicity, I will assume that the singular combines with its nominal argument *apples* before it combines with plural marking.

When the plural function applies to this denotation, it will create groups of grams. Most importantly, if there are enough singularities, it will create groups to which the DP *the 900 grams of apples* can refer. However, despite this promising result, all is not well with the denotation in (8). As would be expected with the previous denotation, sometimes certain groups are too small to be a part of the denotation *grams of apples*. For example consider the sentences in (9).

(9) a. ? Give me 50 grams of apples.
   b. ?? Give me one gram of apples.
   c. ?? 50 grams of apples are rotten.

With the knowledge that apples generally weigh 150 grams, the sentences in (9) sound distinctly odd. For it to be possible to give you 50 grams of apples or one gram of apples, the apples must be exceedingly small. Similarly, they must be small in order for 50 grams of apples to be rotten. In normal circumstances, the phrases *50 grams of apples* and *one gram of apples* cannot be used to refer to anything. This is unexpected if the singular noun has a denotation containing apple bits as suggested in (8). One would expect to be able to use such phrases to refer to bits of apples with the appropriate mass.

In summary, it is difficult to imagine any kind of singular denotation that could consistently combine with the plural function in order to yield a non-empty denotation for *900 grams of apples* but yet yield an empty denotation for *50 grams of apples*.

3.3 Plurals for Quantities Less than One

One final problem for a Link-style analysis of measure nouns comes from that fact that such nouns can appear with numbers that are less than one. Also, when they do appear with such numbers, the measure nouns are requireed to have plural marking. Consider the sentences in (10) and (11).

(10) I bought 0.725 grams/*gram of saffron.

(11) I paid a dollar fifty for the 0.725 grams/*gram of saffron sitting on the table.

The difficulty with such sentences is that the plural function does not create groups ordered below the singularities. The plural function could be amended to create such groups but as I hope to show in section 4, such an arbitrary fix is unnecessary. The solutions for the other two problems mentioned above end up providing a natural explanation for why numbers less than one would require plural morphology.

4. A Unified Semantics of Plurality

Is there a unified interpretation of plurality that not only can provide an adequate analysis of common nouns but can also account for measure nouns? In this section, I attempt to provide such an analysis. My attempt is based on two alterations to the traditional treatment of plural: (1) root (count) nouns, before number marking, are inherently plural and (2) root (count) nouns specify a means of measuring members of the noun's denotation. The technical consequences that follow from these two alterations are three-fold. (i) The plural marker does not change the denotation of the noun, since it is already plural. (ii) The singular marker restricts the noun's denotation to members that measure one unit according to the noun's measurement specifications. (iii) Numeral morphology with respect to a number *n* restricts the noun's denotation to members that measure *n* according to the noun's measurement specifications. Note that these changes to the traditional treatment of plurality do not alter the nature of the phrasal denotations. Noun phrases and sub-phrases still have a lattice-theoretic analysis almost identical to Link (1983). The difference is in how the system calculates and creates these kinds of denotations. In this section, I outline the details of this possible theory of plurality (and number) and explain how it can solve the problems discussed in section 3.

4.1 Details of the Theory

Making nouns underlyingly plural can be implemented in many different ways depending on one's semantic assumptions and preferred frameworks. In what follows, I present one possible implementation of this theory just to give a concrete example. I then discuss the general characteristics of this example that allow it to eliminate the plural function in such a way that correctly addresses the problem of over-generation.

For this particular implementation, I adopt the assumptions of Dynamic Predicate Logic (DPL, Groenendijk & Stokof, 1991) where the influence of an existential quantifier on a variable assignment can reach beyond the quantifier's syntactic scope. In particular, the logic treats the formula schema in (12a) as being semantically equivalent to the formula schema in (12b), where Φ and Ψ represent any formula.

(12) a. $[(\exists n\Phi)\&\Psi]$
  b. $\exists n(\Phi\&\Psi)$

In effect, the interpretation of the existential quantifier treats the quantifier as if it can take scope over an entire conjunction even when syntactically it only takes scope over the first conjunct. This property of Dynamic Predicate Logic has many empirical justifications related to Donkey Sentences and discourse reference. I will not discuss these justification here as they are orthogonal to the present issue (see, Groenendijk & Stokof, 1991 for a discussion). It is sufficient to note here that a dynamic treatment of variable assignment has motivations independent of the present issue.

Given DPL's treatment of existential quantification, consider the following possible interpretation for *pound of potato* without any number specifications.

(13) $[\![$ *pound of potato* $]\!] = \lambda x\exists n(x \in [\![potato]\!]\ \&\ \mu_{lbs}(x) = n)$,
  where μ_{lbs} is a function from entities to the measurement of those entities in terms of pounds.

For simplicity I will treat the noun *pound* as forming a constituent with *of potato*. Number morphology will combine with this entire phrase. However, I doubt that anything crucial depends on this treatment. Also, for now I will assume that $[\![potato]\!]$ simply denotes all the potato-stuff in a given context.

There are two important aspects to this treatment. First, the interpretation in (13) specifies a means of measuring the potato, namely the function μ_{lbs} which measures *stuff* in terms of pounds. Second, the characteristic set[4] associated with the interpretation in (13) contains all the potato stuff that has some mass to it. This will include aggregates of potato that weigh one pound as well as aggregates that weigh three pounds. Since the root noun already contains large aggregates, the plural morpheme no longer needs to do the work of generating groups. In fact, it need not do any work at all. To reflect this in the semantics, the plural morpheme will be interpreted as an identity function that simply passes-up the value of the root noun. This interpretation is given in (14).

(14) $[\![PL]\!] = \lambda P \lambda x(P(x))$

In contrast to the plural morpheme, the singular morpheme must eliminate the large aggregates from the denotation of the noun. In fact, this type of restriction is very similar to the type of restriction imposed by numeral morphemes such as *two* and *three*. To implement this restriction, the singular morpheme and the numeral morphemes will simply specify a value for the measurement variable in the noun. The value will be 1 for the singular morpheme, 2 for *two*, and 3 for *three*. The full interpretations are given in (15).

(15)
a. $[\![two]\!] = \lambda P \lambda x(P(x)\ \&\ n = 2)$
b. $[\![three]\!] = \lambda P \lambda x(P(x)\ \&\ n = 3)$
c. $[\![SG]\!] = \lambda P \lambda x(P(x)\ \&\ n = 1)$

By taking the noun as an argument and then combining it with the number modification through conjunction, the singular morpheme and the numeral morphemes end up altering the denotation of the noun. Due to the properties of DPL, the existentially bound variable in the root noun is modified by the second half of the conjunct in the singular morpheme and the numeral morphemes. For example consider the derivation of *three pounds of potato* given in (16).

(16)

$\lambda x(\exists n(x \in [\![potato]\!]\ \&\ \mu_{lbs}(x) = n)\ \&\ n = 3)$

- $\lambda P \lambda x\ (P(x)\ \&\ n = 3) = [\![three]\!]$
- $\lambda x \exists n(x \in [\![potato]\!]\ \&\ \mu_{lbs}(x) = n)$
  - $\lambda P \lambda x(P(x)) = [\![PL]\!]$
  - $\lambda x \exists n(x \in [\![potato]\!]\ \&\ \mu_{lbs}(x) = n) = [\![pound\ of\ potato]\!]$

[4] In DLP, the characterizing set of a function P, can be give as $\{x : P(x)$ is true with respect to the current variable assignment $i\}$. A formula Φ is true with respect to a variable assignment i iff $\exists h : \langle i,h \rangle \in P(x)$.

The interpretation of *three pounds of potato* is $\lambda x\,(\exists n(x \in [\![potato]\!]\ \&\ \mu_{lbs}(x) = n)\ \&\ n = 3)$. However, due to DPL this interpretation is equivalent to $\lambda x\, \exists n(\,(x \in [\![potato]\!]\ \&\ \mu_{lbs}(x) = n)\ \&\ n = 3)$, where $n = 3$ is within the scope of the existential. As a result, the characteristic set for this noun phrase will contain all and only the potato-stuff in a given context that weighs three pounds.

In summary, there are two important aspects to this proposal. (1) Nouns prior to number marking have the same interpretation as nouns with plural marking: they are interpreted as sets of groups/pluralities. (2) All nouns provide a measure function that can be accessed by the numeral modifiers and the singular morpheme. The singular morpheme and numeral modifiers restrict the denotation of the noun.

4.2 Solving the Problem of Too Many Pounds

The semantics given in the previous section does not generates undesired groups/aggregates. Recall, in situations where there is only a two pound lump of mashed potato on the table, the traditional treatment of plurality allowed the denotation of *three pounds of potato* to be nonempty while also allowing the denotation of *two pounds of potato* to contain more than one member. As a result, the traditional treatment predicts that one should be able to talk about three pounds of potato being on the table. Furthermore, it predicts that one should not be able to refer to the two-pound lump of potato with the phrase *the two pounds of potato*. Both predictions are incorrect.

With the current semantic treatment, this problem disappears. If there is only a two pound lump of potato on the table, then the topmost member in the denotation of $[\![potato]\!]$ only weighs two pounds. There are no members that weigh three pounds and there is only one member that weighs two pounds. In such a situation, the characteristic set of *pounds of potato* ($= \{x :\ \exists n(x \in [\![potato]\!]\ \&\ \mu_{lbs}(x) = n)\}$) is really no different from the denotation of $[\![potato]\!]$. It would contain only one member that weighs two pounds and no members that weigh three pounds. If the numeral *three* restricted this denotation, the characteristic set of the result ($\{x :\ \exists n(x \in [\![potato]\!]\ \&\ \mu_{lbs}(x) = n\ \&\ n = 3)\}$) would be empty. There is no potato-stuff that weighs three pounds in this context. If instead the numeral *two* were to modify the plural denotation, the result would be a denotation restricted to aggregates that weigh only two pounds ($[\![$ *two pounds of potato* $]\!]$ is $\lambda x\,(\exists n(x \in [\![potato]\!]\ \&\ \mu_{lbs}(x) = n)\ \&\ n = 2)$. This denotation would be a singleton set with the the whole potato-lump as its only member. Hence, the uniqueness presupposition of the determiner *the* would be satisfied. In other words, in such a context one could use the phrase *the two pounds of potato* to refer to the two-pound lump on the table.

4.3 Solving the Paradox of Grams

Having shown how the current analysis solves the problem of too many pounds, I will now turn my attention to the what seemed to be a paradox involving grams. Recall that the main source of the problem for the traditional treatment of plurality was that phrases such as *900* *grams of apples* seemed to require that the singular noun *gram of apples* have some kind of non-empty denotation. To have a non-empty denotation would require quantifying over apple bits rather than whole apples. However, if the singular had a nonempty denotation then one should be able to use *30 grams of apples* to talk about bits of apples (or an apple). Yet, unless one is in a context where apples weigh less than 30 grams (a very unusual context indeed), such phrases cannot be used to talk about apples at all. The denotation of *30 grams of apples* seems to require whole apples to weigh less than 30 grams in order for the denotation to be non-empty.

Under the current analysis, there is no conflict between *900 grams of apples* having a nonempty denotation and *30 grams of apples* having an empty one. Since groups are no longer generated by the plural morpheme, there is no need to have singularities in order to get to larger aggregates. Let's consider the current analysis in detail to see why this is so.

In the current analysis, the same type of interpretation can be given to *grams of apples* as *pound of potato*, the only difference would be in terms of the type of measure function and in terms of the nature of the complement noun.

(17) $[\![$ *grams of apples* $]\!] = \lambda x \exists n (x \in [\![apples]\!]\ \&\ \mu_g(x) = n)$

Unlike *potato*, which is a mass noun, *apples* is a count noun. For simplicity, let's assume that this noun denotes the set of all apples and apple-groups.[5] Also, unlike μ_{lbs} which is a measure function that measures entities in terms of pounds, μ_g is a measure function that measure entities in terms of grams.

The characteristic set for $\lambda x \exists n (x \in [\![apples]\!]\ \&\ \mu_g(x) = n)$ is $\{x : \exists n (x \in [\![\text{apples}]\!]\ \&\ \mu_g(x) = n)\}$. This set contains all the apples and groups of apples that weigh n grams for some n. In other words, this characteristic set is no different from the denotation of *apples*. Since apples weigh more than 150 grams, there are no entities in this set that weigh under 150 grams. However, since a group of apples can weigh 900 grams, there remains the possibility that there are groups in the denotation that weigh this amount.

Now given this type of denotation for *grams of apples* in a context containing regular sized apples, the characteristic set of $[\![$*30 grams of apples*$]\!]$ would be empty. The numeral 30 would restrict the characterizing set to things that weigh 30 grams and the result will be an empty set. No apple (or apple group) weighs 30 grams. In contrast, in the same type of context, the characteristic set of $[\![$*900 grams of apples*$]\!]$ need not be empty. The numeral 900 would restrict the characterizing set to things that weigh 900 grams. This set would not necessarily be empty. It is possible to have an apple-group that weighs 900 grams. Since in the current analysis, groups are not created by the plural morpheme from singulars, there is no need to have a non-empty singular denotation in order to have a denotation for *900 grams of apples*.

[5]The analysis of *apples* should be given a more complicated analysis similar to the analysis of measure nouns, however the end result of the more complicated analysis would be similar to the simple assumption adopted here.

4.4 Plural with Numbers Less Than One

In the traditional analysis, there was no explanation of why numbers less than one would require plural morphology. However, in the current analysis one can easily explain this requirement. There are two aspects to this requirement. The first is that numbers less than one can combine with plural nouns. The second is that numbers less than one cannot combine with singular nouns. I address each of the aspects in turn.

Under the current analysis, numbers less than one can combine with plural nouns. This is due to the fact that such nouns have the fewest restrictions on their denotations. For example, consider the following interpretation of *grams of saffron*.

(18) $[\![\textit{ grams of saffron }]\!] = \lambda x \exists n(x \in [\![\textit{saffron}]\!] \ \& \ \mu_g(x) = n)$

This interpretation is no different from the interpretation of the root noun. Note that since the variable n could be any number, including real numbers, the denotation of *grams of saffron* contains entities that weigh less than one as long as the mass noun *saffron* denotes entities that weight less than one. If this denotation were restricted by a numeral such as 0.75, the result would be a denotation that is true of all saffron aggregates that weigh 0.75 grams. In other words, in most contexts that contain saffron, the characteristic set for $[\![$*0.75 grams of saffron*$]\!]$ would be nonempty. Hence, numbers less than one can combine with plural nouns.

Unlike plural nouns, numbers less than one cannot easily combine with singular nouns. To understand why, consider the following derivation of the singular noun *gram of saffron*.

(19) $\lambda x(\exists n(x \in [\![\textit{saffron}]\!] \ \& \ \mu_g(x) = n) \ \& \ n = 1)$

$\lambda P \lambda x(P(x) \ \& \ n = 1)$
$= [\![SG]\!]$

$\lambda x \exists n(x \in [\![\textit{saffron}]\!] \ \& \ \mu_g(x) = n)$
$= [\![\textit{gram of saffron}]\!]$

The resulting interpretation is $\lambda x(\exists n(x \in [\![\textit{saffron}]\!] \ \& \ \mu_g(x) = n) \ \& \ n = 1)$ whose characteristic set would contain all masses of saffron that measured exactly one gram. There are no masses that weigh less than one. If this singular noun were restricted by a numeral such as *0.75*, the resulting function would contain a contradiction and the characteristic set would be a trivially empty.

(20) $\lambda x((\exists n(x \in [\![saffron]\!] \,\&\, \mu_g(x) = n) \,\&\, n = 1) \,\&\, n = 0.75)$

$\lambda P \lambda x(P(x) \,\&\, n = 0.75)$
$= [\![0.75]\!]$

$\lambda x(\exists n(x \in [\![saffron]\!] \,\&\, \mu_g(x) = n) \,\&\, n = 1)$

$\lambda P \lambda x(P(x) \,\&\, n = 1)$
$= [\![SG]\!]$

$\lambda x \exists n(x \in [\![saffron]\!] \,\&\, \mu_g(x) = n)$
$= [\![gram\ of\ saffron]\!]$

The contradiction in this function might be able to explain why such numbers cannot combine with singular nouns.

5. Extending the Analysis

Having shown how the plural is interpreted on measure nouns, it is important to note that the same analysis can be extended to plural marking on common nouns such as *boy*, *man*, *dog*, *cat*, etc. In this section I will use *dog* as a prototypical example of a common count noun. Below, I show how a slight modification in the representation of such nouns will allow them to be integrated into the semantics for plurality discussed in this paper. The goal is to provide the exact same type of denotations for [[*dogs*]], [[*two dogs*]], and [[*one dog*]] as was provided by the traditional Link-style plurality operator, but to arrive at those denotations by different means.

In order to be able to combine with the interpretations of numerals given in the previous section, common nouns must manipulate a variable *n* that is connected to some kind of measure function. Unlike measure phrases such as *pound of potato* and *gram of saffron*, nouns such as *dog* and *cat* are not associated with measurements in terms of weight. However, there is a sense in which they are associated with measurements in terms of the number of atoms within a particular group – a measurement that is similar to set cardinality. We use phrases such as *two dogs* to talk about and quantify over groups of dogs with two members. Similarly, we use phrases such *two cats* to talk about and quantify over groups of cats with two members. Let's use the symbol *AT* to represent the function that maps groups to a measurement (a natural number) that is equal to the number of members in that group. Thus $AT(\{a,b,c\})$ will be equal to 3 no matter what the nature of the members of the group are. They could be dogs, cats or even abstract objects such as ideas.

With this measure function in mind, the interpretation of *dog* can be represented in a way that is similar to the interpretation of *pound of potato*. Consider (21).

(21) $[\![\textit{dog}]\!] = \lambda x \exists n(DOG(x) \,\&\, AT(x) = n)$

The symbol *DOG* represents the concept of doghood, a concept that is true of all dogs and all groups of dogs. The measure function in this denotation is the one that measures the cardinality of groups. The characteristic set for (21) is the set of all dogs and dog groups. The plural morpheme does not change the interpretation: the plural noun *dogs* has the same interpretation as the root noun *dog*.

If this denotation were restricted by a numeral such as *two*, the resulting characteristic set would contain all the groups whose cardinality is two. This is no different than the resulting denotation in a Link-style analysis. If the denotation of the root noun were to combine with a singular morpheme, the resulting denotation would contain all the groups whose cardinality is one. In other words, the set of singular dogs. Once again, the resulting denotation is no different than the one provided by the traditional analysis.

In summary, not only can the analysis of plurals and numerals proposed in this paper account for measure nouns, it can also be extended to account for common nouns.

6. Conclusion

In this paper, I have demonstrated that the traditional treatment of plurality as a group-forming operation encounters three problems with respect to measure nouns. It generates groups that are empirical unsubstantiated. It fails to generate large groups from empty singular denotations even though empirically this seems to be required. It cannot explain why numerals less than one must combine with plural nouns. By interpreting plural as an identity function and in contrast interpreting root nouns as inherently plural, all three problems can be over-come. This new treatment of plurality easily extends to all common count nouns.

References

Bale, A., and D. Barner. inpress. The interpretation of functional heads: Exploring the mass/count distinction. *Journal of Semantics* .

Borer, H. 2005. *Structuring sense*. Oxford: Oxford University Press.

Bunt, H. C. 1979. Ensembles and the formal semantics properties of mass terms. In *Mass terms: Philosophical problems*, ed. J. Pelletier, 249–277. Dordrecht: Reidel.

Bunt, H. C. 1985. *Mass terms and model-theoretic semantics*. New York: Cambridge University Press.

Chierchia, G. 1998. Plurality of mass nouns and the notion of "semantic parameter". In *Events and grammar*, ed. S. Rothstein, 53–103. Dordrecht: Kluwer.

Gillon, B. 1992. Towards a common semantics for english count and mass nouns. *Linguistics and Philosophy* 15:597–640.

Gillon, B. 1999. The lexical semantics of english count and mass nouns. In *The breadth and depth of semantic lexicons*, ed. E. Viegas, 19–37. Dordrecht: Kluwer.

Groenendijk, J., and M. Stokof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14:39–100.

Krifka, M. 1995. Common nouns: A contrastive analysis of chinese and english. In *The generic book*, ed. G. Carlson and J. Pelletier, 75–116. Chicago: U. of Chicago Press.

Lasersohn, P. 1995. *Plurality, conjunction and events*. Dordrecht: Kluwer.

Link, G. 1983. The logical analysis of plurals and mass terms: A lattice-theoretical approach. In *Meaning, use, and interpretation of language*, ed. R. Bauerle, C. Schwarze, and A. von Stechow, 302–323. Berlin: de Gruyter.

Link, G. 1998. *Algebraic semantics in language and philosophy*. Stanford: Center for the Study of Language and Information.

Quine, W. V. O. 1963. *Set theory and its logic*. Cambridge: Harvard University Press.

Sauerland, U. 2003. A new semantics for number. In *Proceedings of SALT XIII*. Cornell, Ithaca: CLC Publications.

Sauerland, U., J. Andersen, and K. Yatsushiro. 2005. The plural is semantically unmarked. In *Linguistic evidence*, ed. S. Kepser and M. Reis. Berlin: Mouton de Gruyter.

Schwarzschild, R. 2002. The grammar of measurement. In *Proceedings of SALT XIII*. Cornell, Ithaca: CLC Publications.

Selkirk, E. 1977. Some remarks on noun phrase structure. In *The generic book*, ed. P. Culicover, T. Wasow, and A. Akmajian, 285–316. New York: Academic Press.

Spector, B. 2003. Plural indefinite dps as plural-polarity items. In *Romance languages and linguistic theory 2001*, ed. J. Quer, J. Schroten, M. Scorretti, P. Sleeman, and E. Verheugd, 295–313. Amsterdam: John Benjamins.

Spector, B. 2007. Aspects of the pragmatics of plural morphology: On higher-order implicatures. In *Presuppositions and implicatures in compositional semantics*, ed. U. Sauerland and P. Statev, 243–281. Palgrave-Macmillan.

Alan Clinton Bale
CMLL, Concordia University
1455 de Maisonneuve Blvd. W., H663
Montréal, QC (H3G 1M8)

alancbale@gmail.com